





REMARKS ON (a) TOWARDS A NORMATIVE THEORY OF ORGANIZATION DESIGN CONTROL and (b) A GAME THEORETIC ACCOUNT OF SOCIAL JUSTICE?

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The Rand Corporation Santa Monical California 90406 REMARKS ON (a) TOWARDS A NORMATIVE THEORY OF ORGANIZATION DESIGN CONTROL and *
(b) A GAME THEORETIC ACCOUNT OF SOCIAL JUSTICE

Alain A. Lewis

*S.R.I. Discussion Papers, W. H. Brock, 1978

This paper was written at Harvard University for a Project on Efficiency in Decision Making.

ABSTRACT

A critical review of two papers in the subject of The Theory of Games by W. H. Brock of S.R.I. is provided. The framework of comparison employs the Hempel concept of a formal model (qua theory) to construct a partial order on deductive explanans - explandum schemata. In addition, a comparison is made between the works in question and existing theory on the subject.

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My comments concerning the works in question are directed along two lines. Firstly, I find a singular omission of substantive comparison with existing results concerning the same issues addressed. Secondly, I feel that the limitations to judicious use of the particular approach taken to define the underlying Game Theoretic structures are not adequately spelled out in the context of the issues with which they attempt to deal. It is therefore difficult to ascertain from a critical standpoint, the relative merits of these works as improvements in a model theoretic sense, to existing theory.

Before engaging in more detail, I should like to make transparent what is meant by an improvement in a model theoretic sense. Consider a model to be in the widely accepted sense of Hempel[1] a deductive scheme:

$$M(\alpha) = \frac{C_1^{\alpha}(p), \dots, C_k^{\alpha}(p)}{M_1^{\alpha}, \dots, M_k^{\alpha}}$$
 Explanans
$$E_1^{\alpha}, \dots, E_m^{\alpha}$$
 Explanandum

A model M(α) consists of a class of premises with interpretations within a well defined phenomelogical context $P_{,}^{\alpha}$ C_{1} (p), ..., C_{k}^{α} (p), a class of rules which specify the formulae constructed by arrangements (perhaps with exogenous parameters to be precise), $(M_{1}^{\alpha}, \ldots, M_{k}^{\alpha})$, and a class of expressive desiderata, or theorems, $(E_{1}^{\alpha}, \ldots, E_{m}^{\alpha})$. Conceivably, one can have a class of models $M(\alpha_{j})$ $j=1,\ldots,m$ relative to a fixed phenomelogical context.

Construct an ordering on the $M(\alpha_{i})$ as follows:

$$M(\alpha_i) \gtrsim M(\alpha_i)$$
 i = j if either

(I)
$$\left\{ C(p)_{j} \right\}_{M(\alpha_{i})} \subseteq \left\{ C(p) \right\}_{j=M(\alpha_{j})}$$

and

$${E_k}_{M(\alpha_i)} \equiv {E_k}_{M(\alpha_i)}$$

with

$$\left\{^{M}\right\}_{M\,(\alpha\,\,\mathbf{i})}\,\,\,^{\textcircled{\tiny 0}}\,\,\left\{^{M}_{k}\right\}_{M\,(\alpha_{\,\,\dot{\mathbf{j}}})}$$

where $\{\cdot\}_{M(\alpha \, i)}$ refers to the collection of concepts specific to $M_{(\alpha \, i)}$ and \oplus denotes independence of comparability (possibly non-comparable). This we denote as the Weak Efficiency Criterion for model comparison.

(II)
$$\left\{ C(p) \right\}_{M(\alpha i)} = \left\{ C(p) \right\}_{j = M(\alpha j)}$$
and
$$\left\{ E_{k} \right\}_{M(\alpha i)} \stackrel{\triangleright}{+} \left\{ K_{k} \right\}_{M(\alpha j)}$$
with
$$\left\{ M_{k} \right\}_{M(\alpha i)} \stackrel{\Phi}{\Phi} \left\{ M_{k} \right\}_{M(\alpha j)}$$

This we denote as the Weak Semantic or Weak Expressive Criteria for Model Comparison. (A model can be weakly superior to another i.e., $M(\alpha \, i) \not \geq M(\alpha \, j)$ if it is both weakly more efficient and weakly more expressive.)

The use of "weak" is in reference to the lack of need for comparison of rules in defining the ordering.

If one can make comparisons of rules as well then the above criteria can be instantiated to read Strongly Efficient or Strongly Expressive as,

$${M_k}_{M(\alpha i)} \stackrel{\epsilon}{+} {M_k}_{M(\alpha j)}$$

Thus a partial order (possibly strict) can be placed upon a class of models in the context of a well defined class of phenomena for which deductive statements have interpretations (are K satisfiable, or simply true in two-valued logic) if either more efficiency in terms of fewer premises required for the same class of expressive desiderata can be achieved, or if more expressive desiderata can be obtained for a given set of premisses. The strengthened version of the ordering captures the aesthetic sense of mathematical elegance involving the use of fewer steps of reasoning to arrive at the same result.

The feature of the above scheme is that it may well be the case that

$$\sim \left(\begin{smallmatrix} M_{(\alpha i)} & \gtrsim & M_{(\alpha j)} \\ & \text{and} \\ & \sim \left(\begin{smallmatrix} M_{(\alpha j)} & \gtrsim & M_{(\alpha i)} \\ & \gtrsim & M_{(\alpha i)} \\ \end{smallmatrix}\right)$$

In which case the relative worth of the models in comparison to each other must be made on some other basis. In either case, it is essential to be able to make these distinctions in a precise manner in the context of critiques of theories, comparison of theories, or contributions to theories, since otherwise one finds oneself very quickly in the realm of excessively general discourse.

For example, in a Game Theoretic Model, included $\inf\{C_K\}_{M(\alpha j)}$ would be assumptions of the number of participants, rationality of the participants, informational characteristics, centralized or decentralized structure of the environment, and participant structure in terms of coalitional alignments. Included in $\inf\{K\}_{M(\alpha j)}$ would be the rules of the game, i.e., maximizing pay off functions in a static context, and in the context of a dynamic setting, adjustment procedures through time. Included in $\inf\{E_K\}_{M(\alpha j)}$ would be concepts like optimal solutions, equilibrium solutions, equitable solutions, stable solutions, self-enforcing solutions, regulated solutions, incentive compatible solutions, etc.

We note that our sense of model is intended to generally formalize the notion of theory and as such may or may not be causal since this latter concept involves verifiability and to some extent a high degree of inductive confirmation. The point is that causal schemes are at least deductive.

As a concrete example of the foregoing method of model comparison consider a game played in a market of exchange by participants P. One might ask, What is the minimal cardinality of the size of participants that will yield the equivalence between the Core and Competitive equilibrium, the former being a cooperative game solution concept, and the latter being a non-cooperative game solution concept?

It is established that when the size of P, |P|, is that of the continuum, Core \equiv C.E. But can this be achieved with a lesser number of participants, say N_0 ? A model which can express the above equivalence with the assumption of $P = N_0$ instead of $|P| = 2^{N_0}$ is more efficient by our criterion. If,

in addition, such a model can find expression for new relationships that are not expressible by the other, it becomes superior, in the weak sense, to the other. That is, if Core \equiv C.E. and Core \cap $M^{i}(x) \neq \phi$ for $|P| = \bigcup_{i=1}^{N} Q_{i}$, but it is not expressible that \exists $M^{i}(x) = 0$ for $|P| = 2^{NO}$, the model with $|P| = \bigcup_{i=1}^{N} Q_{i}$ is superior in the weak sense.

It is in the above sense of model comparison, that we feel the author's work in q estion is deficient, as it does not allow comparative evaluation with existing theory.

With reference to the first work in question, the author begins with a taxonomy of problems which arise in Normative Organizational Theory. (pp. 2-3)

Item (I) It is true that "Pareto-Dominance" applied to outcomes and mechanisms of choice and decision making gives at best a partial comparison of what is optimal as can be confirmed in any Micro-Economics Text on Welfare Economics. But the inference that Nash Equilibria therefore are indeterminate does not follow, since it links uniqueness with determinacy, which must either be assumed or demonstrated. Equilibria of Competitive Markets are unique under certain conditions (as are Nash Equilibrium by the way) but that there exists equilibrium prices, and in the Nash context equilibrium strategies, well defined within the context of decentralized maximizing behavior, is certainly true even though the conditions of Weak Gross Substitutes in C.E. or diagonal strict concavity in N.E. may not be met in order to insure unique outcomes.

It is further asserted that a theory of Harsanyi's identifies a particular point in Nash outcomes that is most stable and therefore most optimal. But stable in what sense? There is the failure here to make distinctions between static concepts and dynamic ones, i.e., equilibrium, optimality and dynamic mechanisms within which question of stability can be posed and answered. Even then there is no necessary coincidence between stability and optimality. Granted, both are desirable features, but they are distinct.

There is further reference to a "deeper problem" that is conceived of as being a basic incompatibility between decentralized rational behavior and the normative issues formulated in cooperative games. However, we are left unenlightened as to what these concepts are, in fact, since these key terms are left entirely unspecified. The comment also completely ignores the monumental contributions in Mathematical Economics to Game Theory demonstrating the conceptual equivalence between the Core, a cooperative Game Theoretic solution which strictly includes Pareto Dominance and Competitive Equilibrium, a decentralized rational choice scheme, in the context of large Neo-Classical economies.

On this last item, it should be further noted that significant work on the notions of "fairness" and "equity" and more importantly "envy" (which the author does not mention) is found in the works of Schmeidler and Vind[2] and Hal Varian[3], relating these notions to Competitive Equilibrium and therefore to the Core in cooperative Game Theory, which we shall deal with in more detail below.

It is clear that some comparison between the concepts of Bargaining Equilibrium as employed by the author and the concepts of Competitive Equilibrium and other cooperative game solution concepts is needed in substance here before the "depth" of the problem can be ascertained. (I have in

mind specific comparisons between the Kernel, Bargaining Set, Nucleolus, ϵ -Core, ϵ -Bargaining Set, and etc.)

There are several minor issues which should be commented on. On Page 4, a confusion between preferences and utilities is apparent. Generally, preferences are considered more fundamental, and in fact they cannot always be represented by utilities. In fact, very often, representation of preferences as utilities, apart from work in foundations, becomes the major problem in Game Theoretic analysis.

The meaning of redundant (as applied to the author's set of norms) needs to be specified, i.e., do all these norms each imply all the rest? And what is the meaning of the "game-theoretic-sense-of-stability" out of the context of any mention of dynamics?

I find here no mention of existing works in Game Theory concerning the normative evaluation of outcomes; specifically the equal treatment properties of the Core, Competitive Equilibrium, and the Kernel in the Aumann-Maschler scheme of Games.

Which brings the point of how does this formulation compare to some of the empirical results obtained by Maschler on the Bargaining Set where rational behavior can be observed and correlated by experiment? That is to say, what are the advantages of implementation of this scheme, since it must make utility comparisons at conceptual levels but not operationally, and how are these contexts characterized?

There is also mention of a previous work concerning an unbiased representative decision procedure. In fairness, I

have also read the author's paper on this subject critically and find the following two deficiencies:

- (1) The composition of Games employing G. Owen's procedure is mathematically correct but theoretically uninteresting since the component games are inessential, which class G. Owen himself cites as not exhibiting any coalitional properties since participants are just as well off playing by themselves. What is the reason here to form any coalition? The issue is, the composition of the essential case, since the raison d'être for Cooperative Games is coalescence.
- (2) While the Selten value is useful as a generalized Shapley value, it is still an expectation and the outcomes thus generated are expectations of outcomes, not the outcomes themselves. It is known that additional conditions must be imposed in competitive markets in order for Ex-Ante equilibria that are optimal to coincide with Ex-Post allocations that are optimal. The difference being that between anticipated trades and actual ones. I find no such criteria here concerning the analysis situation with the Selten Value as applied. How is it possible for one to play an expectation? Rather one bases actual play in accordance to the acceptibility of the expectations of anticipated play.

Harsanyi clearly recognizes this when he refers to "Dead-lock" games (c.f. "Rationality Postulates..." M. Sci. 1961).

Mr. 2

Here, Mr. 1. prefers (α_1^1, α_1^2) and Mr. 2 prefers (α_2^1, α_2^2) in terms of their respective pay offs. So that in terms of choice, α_1^1 is dominant for Mr. 1, and α_2^2 is dominant for Mr. 2. There is clearly no cooperation, prima facie, that can take place. And so one says that a mixture of (α_1^1, α_1^2) and (α_2^1, α_2^2) is employed as a maximin alternative but since one of the pairs will result when the chance coin is flipped some explicit acceptance criterion must be either assumed or demonstrated of the expectation vis a vis the outcome. A plausible line of inquiry to this problem is the important work on Bayesian solutions to team games which the author neither references nor compares c.f. Koichi Miyasawa's "Bayesian Approach to Team Decision Problems" [4].

There are yet to more items worth mentioning on the issue of (2): (2)a. The difficulties of bargaining between groups of diverse interests and mistrust, which may very well lead to irreconcilable hostilities, is a further complication that is not at all mentioned by the author. Again, Harsanyi has long since recognized this difficulty, which is a form of uncertainty about other participants, distinct from uncertainty with respect to outcomes. In the work, "Bargaining in Ignorance of the Opponent's Utility Function," J. of Conflict Resolution, 1962.

"The two mechanisms we have discussed are clearly not effective in all cases.

One case where the effectiveness of both mechanisms tends to be greatly reduced is that of bargaining between members of two different societies or cultures. In this case, the two parties are likely to entertain different and mutually inconsistent stereotypes about each other's utility function... thus, both mechanisms... are likely to operate in a much weaker form if they operate at all. In the case of bargaining between communist and non-communist powers all of these difficulties seem to appear in rather extreme form."

Harsanyi goes on to mention a series of sorely needed empirical substantiations of the degree to which stereotypical groups

and their perceptions enter into bargaining and as such make impossible rational outcomes. His remarks (Harsanyi's) can easily be applied to pressure groups, sexual and racial hostilities, economic class envy and etc., all of which are manifest in present times.

(2)b. Closely related to (2)a, is the known possibility that not all coalitions may form, and the use of the uniform distribution as a density in the Selten Value is unwarranted. One must then be concerned with "Quarreling" in groups. There are recent results in the literature addressing this problem that are not mentioned. For example, for two hostile groups, S and Q, what is the increment (possibly negative) in value to a coalition \tilde{S} if $i \in S$ joins given that 1 or more members of Q are present? It must also be mentioned that in such cases a certain strategy-proofness may be required to prevent advantageous misrepresentation of costs and preferences in order to perhaps spite someone else in a different group.

Finally, with regard to the comparative merit of the author's unbiased representative decision making, there is no mention of the seminal contribution of W. H. Riker's, The Theory of Political Coalitions [5] wherein the uniqueness and characterization of various representation weights of strategic protocoalitions* is carried out in great detail within the framework of Cooperative Game Theory. Certainly some brief mention of such a work is required in order to compare the relative merits of the approach taken by the author, especially since Riker's Theory of Protocoalitions is an attempt to deal with essential games, as comprised by endogenous coalition formation.

^{*}A strategic protocoalition is a coalition which an agent may join to effect an impact on the outcome of the group.

Focusing on the second of the two works in question, the author's expression of the belief that "This work makes a significant advance in the foundations of normative organizational theory" (c.f "Towards a Normative Theory of Organizational Design Control" p.7.) must be regarded with much reservation.

My comments here are to be understood in light of the following scheme.

It is my opinion that the second work in question is an attempt to critique without sufficient comparison so that

The first omission of comparison made by the author is concerned with the results of the extended theories of Neo-Classical welfare economics that attempt to deal with issues of envy, equity, and efficiency in the context of the attainable allocations of private ownership exchange economics. Specifically, the work of Hal Varian [3] contains the following account.

Consider a private ownership exchange economy, $(M, \Omega, \{w_j\}_{j=1}^n, \{\ge j\}_{j=1}^m$ for $N = [1, \ldots, n,]$, $\Omega = (R_t)^m$, $w_j \in \Omega$, and $\ge j$ a representable preference ordering on $N \times N$. The sets Ω , and $\{w_j\}_{j=1}^n \subseteq \Omega$, are the commodity space and the profile of initial endowments. An allocation is simply a specification of trade, $x_j \in \Omega$ for each agent $j = 1, \ldots, n$. A feasible allocation is an allocation $x = (x_1, \ldots, x_n)$ such that for prices of m goods p_1, \ldots, p_m $\sum_{i=1}^m p_i x_i \le \sum_{l=1}^m p_i w_l.$ Assume the following definitions to be concerned with feasible allocations.

Df:(1) x is weakly efficient if for x,~ \exists y feasible such that $y_j + j x_j$ for all $j \in \mathbb{N}$.

Df:(2) x is strongly efficient if for x,~ \exists y feasible such that $y_j \gtrsim_j x_j$ for all $j \in \mathbb{N}$ and for some $i \in \mathbb{N}$ $y_i \gtrsim_i x_i$.

Df:(3) x is equitable if and only if $x_i \gtrsim_i x_j$ for all pairs $(i,j) \in x$.

Df:(4) An agent i envies an agent j at allocation x, if $x_j \gtrsim i^{x_i}$.

Df: (5) An allocation x is fair if x is strongly efficient and equitable.

Df:(6) An allocation is weakly fair if x is equitable and weakly efficient.

Theorem: x is fair as an allocation if x is a competitive equilibrium (i.e., feasible and maximal under \geq_j (for all $j \in I$) and if $\sum_{i=1}^m p_i x_{ii} = \sum_{j=1}^m p_j x_{ji}, \text{ for all pairs } (i,j) \in IN \times IN.$

Theorem: If $\left\{ \gtrsim_{j} \right\}_{j=1}^{m}$ is representing and convex, and monotonic, then Ξ x and x is fair.

Df: (7) A coalition is a non-null subset of IN.

Df:(8) If c is a coalition, then c prefers x to y if $x_j \gtrsim_j y_j$ for all $j \in C$.

Df: (9) An allocation is c-fair if no coalition of a fixed size |c| prefers an allocation of a coalition the same size or smaller.

The restriction of Df:(9) to a fixed size is because for a larger coalition, assuming positive endowments, allocations would not be achievable by a smaller group.

Theorem: The only allocations that are c-fair in large economies are those allocations that are competitive equilibrium with equal endowments, i.e., $w_j = w_i$ for all pairs $(j,i) \in \mathbb{N} \times \mathbb{N}$.

In light of the last theorem, the question to be asked of the author's second paper is: Can the distributional scheme proposed be achieved by the price system as a competitive equilibrium? It is also necessary to compare the two notions of fairness employed, for if the author's concept of fairness is an improved notion over that used by Varian and is not achievable by the price system, what modifications are necessary to the competitive mechanism to attain it?

The second omission of comparison, which I take to be more serious than the first, is concerned with the results achieved in the area of constitutional games. This item is central to the subject, for as Professor Harsanyi notes: "The fundamental question of a normative study of social decisions is neither moral nor game theoretic (in the narrow sense). Rather it is a question of organization design, a problem of an optimal constitution: It is how to design social decision making mechanisms so as to achieve certain social objectives or best to satisfy certain value criteria."*

^{*}Bayesian Decision Theory Rule Utilitarianism and Arrow's Possibility Theorem." Discussion Paper, University of Calif., Berkeley, 1978.

The author makes the statement that no satisfactory theory exists for this problem (on pp. 2-3). This comment totally ignores the work done by R. Wilson and S. Bloomfield on Constitutional games of which the following is a brief account.

Consider n participants, [N] = [1,...,n] and a preference order $\{R_j\}_{j\in I}$ defined on I x I where I is a set of issues, assumed to be finite.

Allow a coalition to be a non-null subset of N , and consider a coalitional profile in terms of $\left\{R_j\right\}_{j} \epsilon |N|$.

$$\left\{R_{c}\right\}_{c \in 2} m - 1 = \left\{R_{1}, \ldots, R_{S}, \ldots, R_{m}\right\}$$

Such that R $_{c}$ holds if and only if R $_{j}$ holds for all j ϵ c.

Let there be defined a society preference order R_0 on I x I, then a coalition is said to be effective if $R_S + R_O$.

A constitution is a set of rules which specifies for a well defined class of issues, $C(I) \subseteq I$, which coalitions in N are effective with the respect to items of C(I).

For example: If it is specified that society unanimously prefer an alternative in order to implement it into policy, then the rule of unanimity as a constitution is characterized as being the only effective coalition, i.e., $R_{\rm IN} + R_{\rm O}$. There may be other ways to arrive at $R_{\rm IN}$ by fractional coalescence, and or, vote trading on certain issues, so that the framework is sufficiently general to encompass a variety of methods of decision making, inclusive of bargaining schemes.

Further, Wilson and Bloomfield* derive necessary and sufficient conditions that a constitutional game be representable as a characteristic function game, which is precisely the subject domain of the author. Clearly, the lack of comparison here indicates that the merits of the author's contribution must be judged by criteria other than the existing results and standards of current theory.

^{*}S. Bloomfield and R. Wilson "The Postulates of Game Theory" J. of Math. Sociology 1973.

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